Physics Challenge for Teachers and Students

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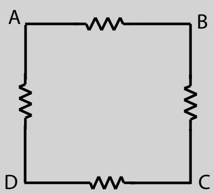
Solution to April 2016 *Challenge*

◗ Back to square one

A circuit below consists of four resistors connected by ideal wires. The circuit draws power *P* if an ideal battery is connected to either points A and D or points B and C. If the same battery is connected to either points A and B or points C and D, the circuit draws power 2*P*.

What power would be drawn if the same battery is connected to points A and C?



Solution:

Let's use the following designations:

$R_{AB} = R_1$

$R_{BC} = R_2$

$R_{CD} = R_3$

$R_{DA} = R_4$

If an ideal battery (ε) is connected to the points A and D, the equivalent resistance is

$$R_{eq1} = \frac{1}{\frac{1}{R_4} + \frac{1}{R_1+R_2+R_3}}. \quad (1)$$

If an ideal battery (ε) is connected to the points B and C, the equivalent resistance is

$$R_{eq2} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_1+R_4+R_3}}. \quad (2)$$

In both cases, the circuit draws power P:

$$P = \frac{\varepsilon^2}{R_{eq}}.$$

So,

$$\frac{\varepsilon^2}{P} = R_{eq1}$$
$$\frac{\varepsilon^2}{P} = R_{eq2}\ .$$

It means that:

$$R_{eq1} = R_{eq2}.$$

Finally, by using Eqs. (1) and (2), we obtain:

$$R_2 = R_4\ . \quad (3)$$

In the same way, if an ideal battery (ε) is connected to the points A and B, the equivalent resistance is

$$R_{eq3} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2+R_3+R_4}}. \quad (4)$$

If an ideal battery (ε) is connected to the points C and D, the equivalent resistance is

$$R_{eq4} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_1+R_2+R_4}}. \quad (5)$$

In that case, the circuit draws power 2P

$$2P = \frac{\varepsilon^2}{R_{eq}}.$$

So,

$$\frac{\varepsilon^2}{2P} = R_{eq3} \quad \text{and} \quad \frac{\varepsilon^2}{2P} = R_{eq4}\ .$$

It means that:

$$R_{eq3} = R_{eq4}\,.$$

Finally, by using Eqs. (4) and (5), we obtain:

$$R_1 = R_3. \quad (6)$$

By using the result of Eqs. (3) and (6) in Eqs. (1) and (2), we obtain:

$$R_{eq1} = R_{eq2} = \frac{1}{\frac{1}{R2} + \frac{1}{R_2+2R_1}} = \frac{\varepsilon^2}{P}. \quad (7)$$

By using the result of Eqs. (3) and (6) in Eqs. (4) and (5), we obtain:

$$R_{eq3} = R_{eq4} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_1+2R_2}} = \frac{\varepsilon^2}{2P}. \quad (8)$$

Taking Eqs. (7) and (8), we obtain:

$$R_2 = R_1(1+\sqrt{3}). \quad (9)$$

By substitution of the result of Eq. (9) in Eq. (8), we obtain:

$$\frac{\varepsilon^2}{P} = \frac{3+2\sqrt{3}}{2+\sqrt{3}} R_1$$
$$\frac{\varepsilon^2}{R_1} = \frac{3+2\sqrt{3}}{2+\sqrt{3}} P. \quad (10)$$

If the same battery is connected to points A and C, the equivalent resistance would be:

$$R_P = \frac{R_1(2+\sqrt{3})}{2}. \quad (11)$$

In that case, the circuit draws power:

$$P_T = \frac{\varepsilon^2}{R_P}. \quad (12)$$

By using Eqs. (10), (11), and (12), we obtain the final solution:

$$P_T = 2(2\sqrt{3}-3)P \approx 0.928\ P.$$

(Submitted by José Antonio Santiago Espinal, student, Escuela Politécnica Superior University of Seville, Seville, Spain)

We also recognize the following successful contributors:

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Guidelines for contributors:

- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address *challenges@aapt.org*. Each message will receive an automatic acknowledgment.
- The subject line of each message should be the same as the name of the solution file (see the instructions below).
- The deadline for submitting the solutions is the last day of the corresponding month.
- We can no longer guarantee that we'll publish every successful solver's name; each month, a representative selection of names will be published, both in print and on the web.
- If your name is—for instance—Donald Duck, please name the file "**Duck16May**" (do not include your first initial) when submitting the May 2016 solution.
- If you have a message for the Column Editor, you may contact him at korsunbo@post.harvard.edu; however, please do not send your solutions to this address.

As always, we look forward to your contributions and hope that they will include not only solutions but also your own *Challenges* that you wish to submit for the column.

Many thanks to all contributors and we hope to hear from many more of you in the future!

–Boris Korsunsky